1. The momentum balance is the same as the falling film of Newtonian fluid in BSL sect 2.2, but the BC at $x=0$ is different. We can use the derivation in BSL Sect 2.2 up to eq. 2.2-9:

$$\tau_{yx} = \rho g x \cos\beta + C_1$$

We apply a shear stress at $x=0$. Then, from eq. 2.2-9, the shear stress at $x=0$ we apply is C_1. $C_1 = \tau_{yx}|_{x=0}$

From Eq. 2.2-9, it is clear that shear stress is a maximum at $x=\delta$. So if the Bingham plastic shears anywhere, it shears at $x=\delta$. So the onset of shearing is when $\tau_{yx}=\tau_0$ at $x=\delta$. Thus

$$\tau_0 = \rho g \delta \cos\beta + \tau_{yx}|_{x=0}$$

or

$$\tau_{yx}|_{x=0} = \tau_0 - \rho g \delta \cos\beta$$

2. Let the inlet surface "1" be just upstream of the tube, and "2" = the outlet. Here there are an abrupt constriction at the inlet*; vertical movement upwards thru 10 m of pipe; kinetic energy out the pipe; + the unknown effect of the fitting. [*constriction might be rounded or sudden. I assume sudden below]

From the macro mech Eng. balance

$$\Delta \tfrac{1}{2} v^2 + g\Delta h + \tfrac{1}{\rho}\Delta P + \tfrac{1}{2} v^2 \tfrac{L}{R_h} f + \underset{\text{(fitting)}}{\tfrac{1}{2} v^2 e_v} + \underset{\text{(constriction)}}{\tfrac{1}{2} v^2 (0.45)} = 0$$

where e_v = friction loss of the fitting,

$$\Delta \tfrac{1}{2} v^2 = \tfrac{1}{2}(10)^2 - 0 = 50$$

$$g\Delta h = (9.8)\,10 = 98$$

$$\tfrac{1}{\rho}\Delta P = \tfrac{1}{1000}\left[1\cdot 10^5 - 7.5\cdot 10^6\right] = -7.4\cdot 10^3 = -7400$$

what is Re? $Re = \frac{Dv\rho}{\mu} = \frac{(0.02)(10)(1000)}{(0.001)} = 2\cdot 10^5$

$k/D \approx 0.004$ Note if $D=0.002$, $R_h = 0.0005$

$f \approx 0.007$

$$\tfrac{1}{2} v^2 (L/R_h) f = \tfrac{1}{2}(10)^2 (10/0.005)(0.007) = 700$$

$$\tfrac{1}{2} v^2 e_v = \tfrac{1}{2}(10)^2 e_v = 50 e_v$$

$$\tfrac{1}{2} v^2 (0.45) = \tfrac{1}{2}(10)^2 (0.45) = 22.5$$

$$\rightarrow 50 + 98 - 7400 + 700 + 50 e_v + 22.5 = 0$$

$$e_v = 6529.5/50 = 130.6$$

[See note on next page]

3. Don't know v or Re, so use trial + error

$Re = Dv\rho/\mu = (0.13)\,v\,(1.26)/(1.75\cdot 10^{-5}) = 9360\,v$

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_s - \rho}{\rho}\right) = \frac{4}{3}\frac{(9.8)0.13}{v^2}\left(\frac{869-1.26}{1.26}\right) = 1170/v^2$$

[what is ρ_s? $\frac{4}{3}\pi R^3 \rho_s = 1\,(kg) = \frac{4}{3}\pi(0.065)^3 \rho_s \rightarrow \rho_s = 869$ as above]

Guess	10 m/s.	$Re = 9.36\cdot 10^4$	from chart, $f \approx 0.5$ →	$v = 48.4$ m/s.
	48.4	$4.5\cdot 10^5$	0.24	69.8
	69.8	$6.5\cdot 10^5$	0.25	68.4
	68.4	(can't see difference in chart)		

$v \approx 68$ m/s

(Actually, this is on the small side for a coconut, but we would have gone off the f(Re) chart.)

For a horizontal pipe, the only fluids that behave like this is the shear-thinning power-law fluid + Bingham plastic

(f) is also correct, or at least possible. Suppose the outlet is higher than the inlet, by 1 m. Suppose $\rho = 1000$ kg/m³.

$$\Delta \mathcal{P} = \Delta p - \rho g (1) = \Delta p - 1000(9.8) = \Delta p = 9800.$$

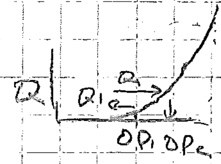
Suppose Δp_1 is 10000 Pa. $\Delta \mathcal{P} = 200$ Pa.
To double Q, all we need is $\Delta \mathcal{P}_2 = 400$ Pa, i.e. $\Delta p_2 = 10200$, much less than $2\Delta p_1$

In more detail

a) $Q \sim (\Delta P)$. $\Delta P_2 = 2\Delta P_1$ NO

b) $Q \sim (\Delta P)^2$. $\Delta P_2 \sim 4\Delta P_1$ NO

c) Could be. If ΔP_1 is just above threshold for flow, then $\Delta P_2 < (2\Delta P_1)$ YES



d) for power-law fluid, $Q \sim (\Delta P)^{1/n}$
for shear-thinning fluid, $n < 1$, $\Delta P_2 < (2\Delta P_1)$ YES

e) for shear-thickening PL fluid, $n > 1$, $\Delta P_2 > (2\Delta P_1)$ NO

f) see above YES

g) opposite of f. $\Delta P_2 > (2\Delta P_1)$ NO

<u>Note on problem 2</u> Several students commented on how large e_v is. It <u>is</u> hard to imagine how a fitting could have such a large value of e_v (90 pairs of sudden contraction followed by sudden expansion? 81 consecutive sharp 90° elbows? Perhaps an <u>almost</u> closed gate valve.) In fact, I made a math error in the problem statement. But this is the correct answer to the problem as stated.